Linewidth Engineering of an Er:fiber Frequency Comb

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***Abstract*—We present a Er:fiber based frequency comb with intrinsic comb linewidths in the order of $1\,\mathrm{kHz}$ at $689\,\mathrm{nm}$ and $813\,\mathrm{nm}$. This opens the way of direct laser cooling of Strontium atoms without the need of locking the frequency comb to optical reference cavities. This was achieved by detailed examination of contributions to the oscillator's phase noise. The oscillator was engineered to be insensitive on pump-noise fluctuations resulting in a low phase noise operation mode of the frequency comb system. The frequency comb generated by the improved oscillator has been proven to show linewidths at $1\,\mathrm{kHz}$ at $689\,\mathrm{nm}$. The stability of the absolute position of the comb tooth is determined by the stability of the Rubidium RF-source used to lock the frequency comb.**

***Index Terms*—frequency comb, linewidth, jitter, fiber optics**

I. Introduction

Integrated quantum clocks exemplify ultracold-atom-based quantum sensors that rely on lasers as a crucial component. Precise control over the quantum states of ions and atoms used in such devices necessitates lasers with narrow linewidths, high spectral stability, and minimal phase noise. To transfer the absolute spectral characteristics to the cooling and trapping lasers, frequency combs come into play. A reduction of the intrinsic linewidths of frequency combs below a few kHz without need of locking to an optical stabilization cavity would simplify quantum clock experiments significantly.

Frequency combs based on mode-locked Er:fiber oscillators exhibit several advantages over solidstate lasers based combs like compactness, alignment-free operation and robustness against environmental influences. State-of-the art systems have typical intrinsic tooth linewidths in the range of several tens of kHz. The broadening of the linewidth is attributed to factors such as pump-induced noise, sensitivity on environmental effects as well as on quantum noise effects. [5] In this work we employ a recently demonstrated technique [2] aiming for intrinsically narrow linewidths in the wavelength range used in an integrated quantum clock experiment based on Strontium atoms ($813\,\mathrm{nm}$, $689\,\mathrm{nm}$).

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II. Theoretical Overview

The spectrum generated by a short-pulse mode-locked laser is defined by the comb equation:

$$f_n = f_{\mathrm{ceo}} + n f_{\mathrm{rep}} \tag{1}$$

Where f_{ceo} is the carrier envelope offset frequency i.e. the frequency of the phase underneath the carrier envelope during subsequent pulse-roundtrips in the oscillator. nf_{rep} is an integer multiple of the the repetition rate.

Several noise sources such as quantum noise, environmental noise and pump induced noise are influencing f_{ceo} as well as f_{rep}. [2], [5] Each noise source has a spectral fix point ν_{fix} where the influence of the noise source on the frequency noise vanishes. [1], [2] According to the elastic tape model the noise contributions scale quadratically with the optical frequency around the fix point.

$$\begin{aligned} S_{\Delta\nu,\nu}(f) = f_{\mathrm{rep}}^2 \Big[& S_{\mathrm{rep}}^{\mathrm{quant}}(f)(\nu-\nu_{\mathrm{c}})^2 \\ & + S_{\mathrm{rep}}^{\mathrm{pump}}(f)(\nu-\nu_{\mathrm{fix,pump}})^2 \\ & + S_{\mathrm{rep}}^{\mathrm{env}}(f)\nu^2 \Big] \end{aligned} \tag{2}$$

$S_{\mathrm{rep}}^{\mathrm{quant}}(f)$, $S_{\mathrm{rep}}^{\mathrm{pump}}(f)$, $S_{\mathrm{rep}}^{\mathrm{env}}(f)$ are the PSDs of the fractional repetition rate fluctuations driven by the quantum, pump induced and environmental noise respectively.

The fix point of the environmental noise leading to variations of the cavity length is located at zero. [2] The influence of the environment on the oscillator is minimized by shielding it mechanically. Quantum noise, steming from ASE features a fix point $\nu_{\mathrm{fix,quant}}$ at the optical carrier frequency $\nu_{\mathrm{c}} = 193\,\mathrm{THz}$. [4], [5]

The pump induced fixed point is given by [2], [4], [5]

$$\nu_{\mathrm{fix,pump}} = \nu_{\mathrm{c}} + f_{\mathrm{rep}}^2 \frac{d\varphi/dP}{df_{\mathrm{rep}}/dP} \tag{3}$$

where φ is the carrier phase. The repetition rate is dependent on the pump power by four mechanisms: [3], [5]

$$\frac{df_{\mathrm{rep}}}{dP} = -f_{\mathrm{rep}}^2 \left(\beta_{2,\mathrm{cav}} \frac{d\omega_{\mathrm{c}}}{dP} + \omega_{\mathrm{rms}} \beta_{3,\mathrm{cav}} \frac{d\omega_{\mathrm{rms}}}{dP} + \frac{\gamma}{\omega_{\mathrm{c}}} \frac{dA^2}{dP} + \frac{1}{\Omega_{\mathrm{g}}} \frac{f_{3\mathrm{dB}}^{\mathrm{Er}}}{f_{3\mathrm{dB}} 2P} \right) \tag{4}$$

The first term in equation (4) is the variation of the group velocity by pump induced spectral shifts of the center frequency ω_{c} governed by the overall intracavity dispersion $\beta_{2,\mathrm{cav}}$. The second term reflects changes of the round trip time with pump power induced fluctuations of the rms spectral width ω_{rms} via the third order dispersion $\beta_{3,\mathrm{cav}}$. The third term is dependent on the peak intensity A^2 of the intracavity pulse and the nonlinearity γ of the fiber oscillator. This is the pump induced contribution to df_{rep}/dP due to self-steepening. The last term reflects the pump induced change of the gain g. In equation (4) dg/dP was approximated by $f_{3\mathrm{dB}}^{\mathrm{Er}}/f_{3\mathrm{dB}}2P$. [5] The sensitivity of f_{rep} on the pump power P can be accessed experimentally by varying the pump power slightly and recording the repetition rate changes introduced by the variations for different pump powers.

The dependeny of carrier offset frequency on the pump power is a consequence of the repetition rate's and the optical phase's variations on the pump power. [3], [5]

$$\frac{df_{\mathrm{ceo}}}{dP} = \frac{\beta_0}{2\pi} \frac{df_{\mathrm{rep}}}{dP} + \frac{f_{\mathrm{rep}}}{2\pi} \frac{d\varphi}{dP} \tag{5}$$

df_{ceo}/dP is experimentally accessable by measuring the changes in f_{ceo} while slightly modulating the pump power. With the experimental values of df_{rep}/dP it is possible to extract $d\varphi/dP$ from equation (5).

$d\varphi/dP$ can also be calculated theoretically using the parameters found in equation (4). For a soliton the sensitivity of the optical phase on pump power fluctuations $d\varphi/dP$ can be calcutated as follows: [3]

$$\frac{d\varphi}{dP} = \frac{\gamma}{4\pi} \frac{dA^2}{dP} \tag{6}$$

According to the elastic tape model the PSDs accross the optical spectrum are connected via equation (2). The linewidth of a comb line can be derived from $S_{\Delta\nu,\nu}(f)$. [4]

$$\delta_\nu = \pi \sqrt{S_{\Delta\nu,\nu}(0) f_{3\mathrm{dB}}} \tag{7}$$

III. Experimental Results

The net-dispersion of the 80-MHz-oscillator cavity was chosen in a way to be able to shift the spectral position of the minimum linewidth in the range of 689 nm (435 THz) to 1560 nm (193 THz) within the mode-lock-window. [2] This was carried out by invesitgating 6 oscillators with differing cavity dispersions. In the following the results for the optimized oscillator are presented.

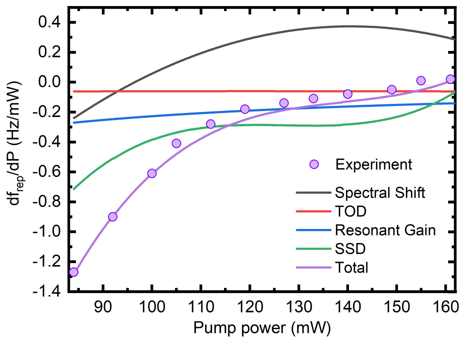

Fig. 1. Experimental data for the pump power dependency of f_{rep} (violet circles). Theoretical values for df_{rep}/dP according to equation (4) are plotted as lines.

Understanding the phase-noise of a fiber-laser frequency comb involves assessing the response of the fiber-oscillator due to changes in pump power. As the spectral position and the linewidth of a comb line is determined by the carrier-envelope offset frequency f_{ceo} and the repetition rate f_{rep} we examined their dependency on pump power independently.

Figure 1 depicts the impact of pump power on f_{rep}. As can be seen the experimental data is perfectly reproduced by equation (4). The sensitivity of df_{rep}/dP on pump-power approaches zero in the region above approximately 120 mW both the experimental as well as the theoretical data show a zero crossing at approximately 160 mW. The pump power was modulated with a square wave amplitude of approximately 2 mW at a few Hz. The resulting changes in f_{ceo} were recorded by a RF-spectrum-analyzer the changes of f_{rep} were recoded by usage of a frequency counter. The f_{ceo} was retrieved from the amplified oscillator output by usage of a f-2f-interferometer. The sensitivity df_{ceo}/dP on pump-power apporaches zero also above approximately 120 mW. It shows a zero-crossing at a pump-power of approximately 160 mW.

The pump induced fix-point-frequency $\nu_{\mathrm{fix,pump}}$ was derived from the experimental results on df_{rep}/dP and df_{ceo}/dP according to equation (3). $d\varphi/dP$ was calculated by inverting equation (5). The results of this calculation are shown in figure 2. The fix points calculated from the experimental data of df_{rep}/dP and df_{ceo}/dP (violet circles) and from the theoretical considerations according to equations (4) and (6) are in remarkable good agreement. The singularity in figure 2 at approximately 160 mW is a result of the zero crossing of the repetition rate sensitivity.

The linewidth of the CEO beat in a f-2-f setup was measured accross the complete modelocking window. From these values the value for $S_{\mathrm{rep}}^{\mathrm{pump}}(0)$ (see equation (2)) was calculated by neglecting $S_{\mathrm{rep}}^{\mathrm{quant}}$ and $S_{\mathrm{rep}}^{\mathrm{env}}$. [2] Combining this result with the derived pump induced fix point frequencies the linewidth at any spectral position can be estimated. As depicted in figure 3 the calculated linewidths at 1550 nm reprocude the exper-

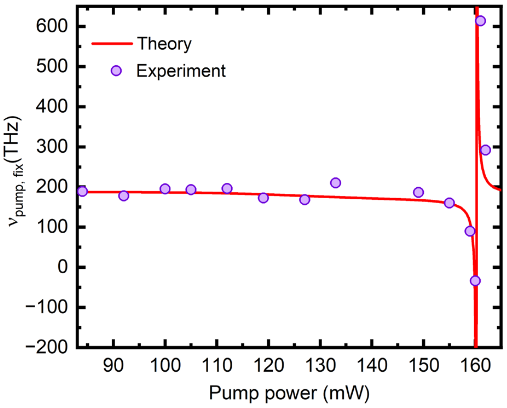

Fig. 2. Fix point vs. pump power of the oscillator. Violet circles: Derivation from df_{rep}/dP and df_{ceo}/dP according to equation (3). Red line: Derivation from equations (4) and (6).

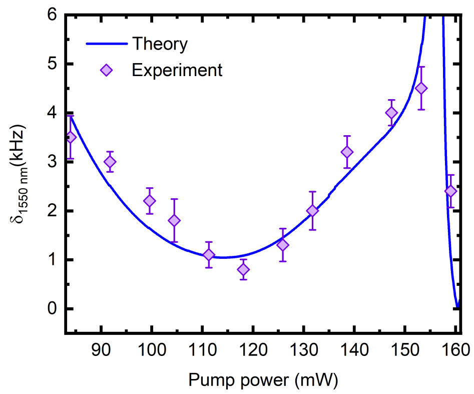

Fig. 3. Linewidth at $1550\,\mathrm{nm}$ vs. pump power of the oscillator.

imentally recoreded linewidths very nicely. The experimental datapoints where recorded by beating the comb output with a laser stablized to an optical high finesse cavity. The beat spectrum at a pump power of $120\,\mathrm{mW}$ is plotted in figure 4. It features a FWHM linewidth as low as $700\,\mathrm{Hz}$.

Nonlinear frequency-conversions extend the frequency comb across a wide spectral range. The method to determine the pump-induced fix point enables the selection of the minimum linewidth position within the output spectrum. [2] It is theoretically possible to the keep the linewidths at $813\,\mathrm{nm}$ and $689\,\mathrm{nm}$ below $2\,\mathrm{kHz}$ simulatneously. To acheive the mimimum linewidths at the wavelengths used in the experment the pump induced fix point $\nu_{\mathrm{fix,pump}}$ has to be shifted to $584\,\mathrm{THz}$. By doing so the linewidth at the fundamental wavelength ($1550\,\mathrm{nm}$) rises up to approximately $3\,\mathrm{kHz}$. The expected linewidths in this range are potentially intrinsically

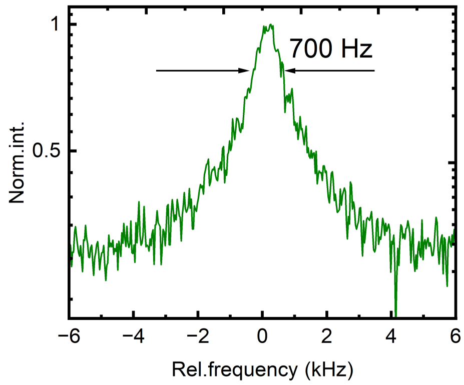

Fig. 4. Measured beat spectrum at $1550\,\mathrm{nm}$ at a pump power of $120\,\mathrm{mW}$.

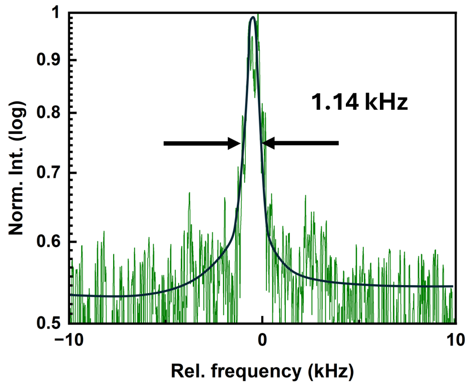

Fig. 5. Measured beat spectrum at $689\,\mathrm{nm}$ at a pump power of $160\,\mathrm{mW}$.

small enough to perform the intended experiment without the need of locking to narrowband optical references. The optical linewidth at $689\,\mathrm{nm}$ was measured at an oscillator pump power of $160\,\mathrm{mW}$, the results are shown in figure 5. The result of the measurement is in remarkable good agreement with the theoretical prediction of $1\,\mathrm{kHz}$ linewidth.

In addition to the linewidth the absolute stability of the comb line is also of gerat importance for a successful application of the frequency comb in laser cooling experiments. To show the comb's absolute stability the RF stabilized comb was beaten with a laser locked to a stabilized optical cavity. The result is depicted in figure 6. The resulting Allan deviation is reproducing the stability of the $10\,\mathrm{MHz}$ reference signal originating from a Rubidium clock. This results in an absolute stability of the comb line at $689\,\mathrm{nm}$ of approximately $100\,\mathrm{kHz}$. First measurements with the Dutch White Rabbit $10\,\mathrm{MHz}$ RF reference [6] already show a stability imprpovement of one order of magnitude compared to the Rubidium clock referenced setup. Detailed measurements with the improved RF reference

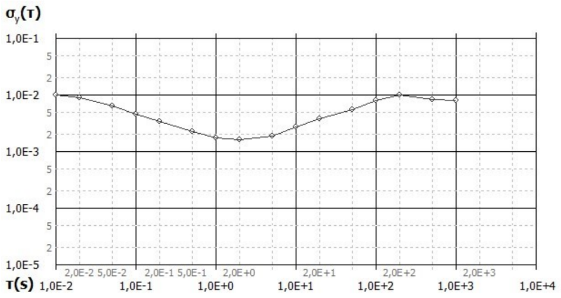

Fig. 6. Allan deviation of a $10\,\mathrm{MHz}$ beat at $689\,\mathrm{nm}$ of the Rubidium clock referenced comb with a stabilized optical cavity.

are subject to current investigations. With this reference the absolute stability of the passively narrow comb teeth is in the range of $10\,\mathrm{kHz}$ providing a significant improvement in the ease of use of laser cooling applications.

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